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A NETWORK FLOW MODEL FOR THE VEHICLE ROUTING PROBLEM WITH TIME WINDOWS AND MULTIPLE ROUTES

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INTRODUCTION

The Vehicle Routing Problem (VRP) is a combinatorial optimization problem that has been widely studied in the literature, ever since it was formulated for the first time in (Dantzig and Ramser 1959). It can be seen as a generalization of another well know combinatorial problem, the traveling salesman problem, which can be described as a VRP with one vehicle, no depot, no vehicle capacities and no customer demands. Generally speaking, it is the problem of scheduling a fleet of vehicles to visit a set of customers, to whom they must deliver or collect a demanded quantity of goods. The problem consists of finding the best set of routes, according to a given objective function, such that all operational constraints of the vehicles are respected, and the set of customers is covered. This objective function can be the minimization of all traveling costs, the maximization of the number of served customers, or some combination of these or other factors. The VRP is well-know to be NP-hard, and so are most of its variants. Its solution methods include several heuristic and metaheuristic approaches, as well as some exact methods, mainly based on branch-and-bound techniques. The classical version of the VRP is commonly called the Capacitated Vehicle Routing Problem, as the vehicles in the fleet have limited capacities. There are several variants of this problem. In (Toth and Vigo 2002; Cordeau, J.-F et al. 2007), the authors describe the VRP and some of its main variants.

We addressed a variant of the classical vehicle routing problem, the Vehicle Routing Problem with Time Windows and Multiple Routes (MVRPTW). Despite its apparent practical relevance (delivering perishable goods, for example), this variant of the classical VRP has not been the subject of a large number of studies. It considers that a same vehicle can be assigned with more than one route per planning period. This means that it can perform one route, reload at the depot and leave to the following route until the end of the workday. In addition, all customers must be served within a predefined time period that corresponds to their time windows.

SOLUTION METHOD

We developed a new network flow model, and an exact solution algorithm to solve this problem. In the network flow model, every workday (sequence of routes assigned to one vehicle for one planning period) corresponds to a path in an acyclic directed graph. It is formulated as a minimum flow problem, whose nodes correspond to time instants of one workday and whose variables correspond to feasible routes.

All vehicle routes are previously generated. The integer model is then solved with a commercial software (CPLEX), explicitly considering all its variables. Variables represent feasible vehicle routes in the network flow model. Reducing the number of arcs reduces the size of the model, increasing its efficiency. We derived four propositions defining dominance rules that allow us to discard some routes that are never interesting when compared to some other one. With them, we were able to reduce approximately 29% of variables to include in the model.

A route is defined by a sequence of visits to a subset of customers. A vehicle route is feasible if the sum of the demands of all customers served by it does not exceed the vehicle capacity and if its sequence of visits is such that it is possible to visit every customer within its time window. In addition, we also consider that the service of all customers in the route cannot star later that t_{max} time units after the route begins. It may be possible to begin a route at different time instants, keeping it feasible. Therefore, for every route r, we consider that there are several routes r_t, one for each possible departure instant t. The duration of a route may be different for different departure instants, as the waiting times to serve customers, due to their time windows constraints, may vary.

As the nodes of the graph represent time instants, a discretization of time is required. We do it by using some rounding procedures. We embedded our model in an algorithm that allows us to correct some problems that may occur due to those rounding procedures. This algorithm ensures that our solution method is exact. It consists of disaggregating some nodes of our graph whenever it is necessary. This means that we go back and consider the real values of some eventually conflicting arc extremities in order to correct the solution. However, this disaggregation is not always necessary, even if the solution proves to be unfeasible. Given that a same route can begin at different time instants, before going to the more computationally expensive procedure of disaggregating nodes, we can verify if it is possible to get a feasible similar solution with the same sequence of routes in the workday, but with different beginning instants for some of those routes. This new solution would have the same cost, and thus, would also be an optimal solution.

RESULTS AND CONCLUSIONS

To our knowledge, the only other exact approach to this variant proposed in the literature is a branch-and-price algorithm (Azi et al. 2010). The master problem of the column generation scheme is a set covering model, where each column represents a vehicle's workday. The pricing problem is an elementary shortest path problem with resource constraints, formulated in a graph whose nodes represent vehicle routes, which are also generated *a priori*.

We conducted some computational experiments on a set of benchmark instances and compared our algorithm with the column-generation based algorithm described in (Azi et al. 2010). Our method outperforms it in many cases. In the cases where the two methods find a solution, our method dramatically reduces the computational time needed. This reduction is around 95%.

REFERENCES

Dantzig, G. B. and J. H. Ramser. 1959. "The Truck Dispatching Problem". *Management Science* 6 No.1, 80-91.

Toth, Paolo and D. Vigo. 2002. "The Vehicle Routing Problem". *SIAM Monographs on Discrete Mathematics and Applications* 9.

Cordeau , J.-F., G. Laporte, M. W. P. Savelsbergh and D. Vigo. 2007. "Vehicle Routing". In *Transportation, Handbooks in Operations Research and Management Science*, C. Barnhart and G. Laporte (Eds.). 367-428.

Azi, Nabila, M. Gendreau and J.-Y. Potvin. 2010. "An exact algorithm for a vehicle routing problem with time windows and multiple use of vehicles". *European Journal of Operational Research* 202, 756-763.

AUTHOR BIOGRAPHIES

RITA MACEDO has graduated in Mathematics and Applied Mathematics in 2003 and 2007, respectively, at University of Minho. Since 2008 she has been working on her PhD thesis, in Operations Research, at University of Minho.

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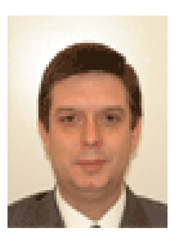

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